THE PROBABILISTIC THINKING OF PRIMARY SCHOOL PUPILS IN CYPRUS: THE CASE OF TREE DIAGRAMS

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In this research work we explored the nature of 9-12 year old pupils' responses to probabilistic problems with tree diagrams. It was found that a large percentage of pupils failed to respond correctly even to very simple problems that demanded the identification of 'possible routes/paths' in figures with tree diagrams/mazes. The results also revealed the existence of subjective elements and other errors in pupils' thinking. The data were generated in year 2000 when the new mathematics books were introduced extensively in the primary schools. Comparisons are done by age and gender. The results of the study form a general overview and build the basis for further and more focused research because the relevant literature, especially regarding Cyprus, is very sparse.

INTRODUCTION

The probabilistic thinking of primary school pupils has been the center of much research during the last decades. One could claim that Piaget and Inhelder (1975) initiated a vast amount of research into pupils' capacity to compare two probabilities. Other researchers followed and studied pupils' probabilistic thinking (Green, 1983; Fischbein & Gazit, 1984; Kapadia & Borovnik, 1991; Canizares, Batareno, Serrano & Ortiz, 1997).

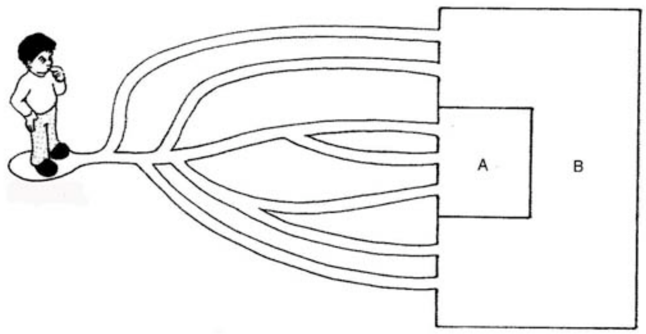
The identification of errors and misconceptions of pupils are of paramount importance in order for the teachers to maximize their teaching effectiveness. To this end, much of the recents research has focused on specific probabilistic errors and misconceptions (Afantiti Lamprianou & Williams, 2002; Alatorre, 2002; Lamprianou & Afantiti Lamprianou, 2002; Kafoussi, 2003; Lamprianou & Afantiti Lamprianou, 2003).

Not all the areas have, however, been researched adequately. One of the relatively under-researched areas, especially in the context of the primary schools in Cyprus, is the way the pupils think and respond when confronted with questions that demand interpretation of tree diagrams (e.g. find possible routes in a maze) and multiplication of probabilities.

Green (1987) described a situation where pupils were confronted with such a question, which asked them to follow a robot's path through junctions of a maze and decide at which exit the robot was most likely to end up. Green mentioned that the question had been designed to be difficult for the younger pupils but easier for the older pupils who had met tree diagrams and multiplication of probabilities before. Green said

> However, the results of large scale testing showed that the item was extremely difficult for *all* [italics his] pupils … Interviews conducted with individual pupils indicated that although they [the pupils] had studied tree diagrams they could not see the relevance of that work to the problem posed. (p.157.)

Ojeda (1999) also used similar questions. In an experiment, marbles were allowed to roll free through a maze with several exits. Pupils were asked to identify the exit from which most of the marbles would come out. Others were asked to identify the proportion of marbles that would come out of each of the exits. Ojeda found that many pupils failed to understand how the maze worked and failed to understand issues of equiprobability e.g. at a junction, a ball has 50% chance of going right or left.



1. What is the probability of the boy entering room A? …………………..
2. What is the probability of the boy entering room B? …………………...

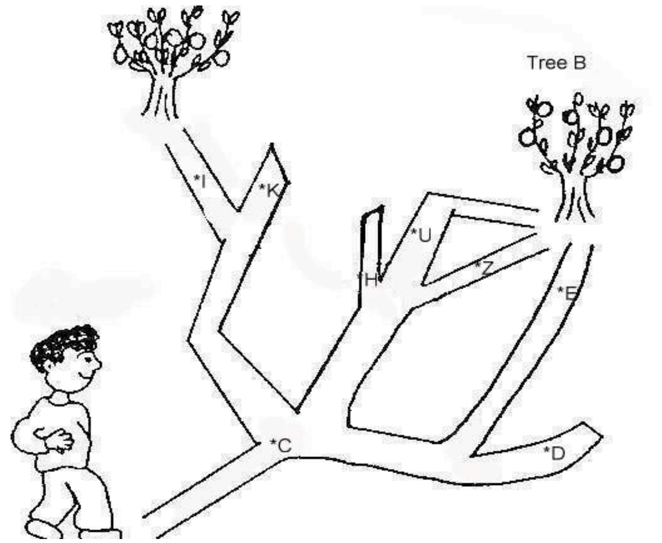
Figure 1: A sample of the 'tree diagram' questions of the 'new' mathematics books

Questions very similar to the one mentioned by Green are included in the recently introduced 5th Grade mathematics books for Cyprus primary schools (see Figure 1). The 5th Grade pupils of this study had therefore met this type of questions before. The 6th Grade pupils had never been confronted with this type of question, however, since such questions did not appear in their books (at the time of the study they had not yet received the new version of the mathematics books). The 4th Grade pupils are also assumed to be largely unfamiliar with these types of questions because such questions do not appear in the revised 4th Grade books.

However, no published research exists that investigates the way the pupils in Cyprus think when attempting such probability questions with tree diagrams. The relevant literature at the international level is also relatively sparse. This research is important because it investigates an under-researched topic in the context of the primary education in Cyprus. This topic is also relatively under-researched at the international level as well.

AIMS

This study explores the probabilistic thinking of primary school pupils in Cyprus aged between 9-12 years old when attempting questions involving tree diagrams. It also aims to study the effect of grade, age and gender on pupils' thinking. In addition, the research studies the extent to which subjective elements affect the pupils' thinking.



Tony wants to pick oranges either from tree A or from tree B. He can choose any one of the routes to reach either tree A or tree B. What are the possible routes to tree A? (please write all the possible routes). What are the possible routes to tree B? (please write all the possible routes). What is most likely, to reach tree A or tree B? Why?

Figure 2: Example of a question of the test

METHOD AND INSTRUMENT

The research instrument (a test) was developed through a series of pilot steps. The final version of the test consisted of five questions. A sample of the questions is given in Figure 2. The question in Figure 1 also appeared in the test with very slight changes.

As is indicated in Figure 2, each question consisted of several other questions (except from question 1 which had no sub-questions). Overall, the questions may be classified into the four groups displayed in Table 1.

	Type of Questions	N of Questions
1	Indicate the possible routes (e.g. What are the possible routes?)	8
2	Indicate the most likely outcome (e.g. What is most likely, to reach tree A or tree B?)	4
3	Indicate the probability (e.g. What is the likelihood to reach tree A?)	4
4	Explanation (e.g. Why?)	1
	Total number of questions	*17*

Table 1: A classification of the test questions

This table indicates that the test may be divided into three major scales which test (a) the ability of the pupils to 'read' and interpret tree diagrams and find possible routes (questions of category 1); (b) the ability of the pupils to indicate the most likely between two possible outcomes taking into account the possible routes for each one (category 2); and (c) the ability of the pupils to compute the probability of events to happen (category

3). Finally, an 'Explain Why' question forms a fourth category but cannot really be named as 'scale' as it consists of only one question. Each correct response is awarded a mark. Therefore, the number of questions in each scale/group also represents the number of maximum available marks.

In the analysis, the performance of the pupils in the four groups of questions, hence scales, will be considered separately because each of the scales measures a different skill. It is also meaningful to investigate the relationship of the demographic variables of the pupils with their performance on each of the scales mentioned in Table 1.

THE SAMPLE

The final instrument was administered to 776 pupils in four different randomly selected district schools in Cyprus. Approximately 50% of the sample were boys (N=391), one pupil did not specify his/her gender and the rest were girls (N=384). The sample consisted of 264 pupils in Year 4 (34%), 242 pupils in Year 5 (31.2%) and 270 pupils in Year 6 (34.8%).

THE RESULTS

The results of the research will be presented in two sections. First, the results will be discussed briefly, without any reference to demographic variables (e.g. gender, grade or age). Then, a more elaborate analysis will follow with reference to the demographic variables of the pupils.

Overview of the results

Questions of the first scale (What are the possible routes?):

A large number of pupils failed to identify the possible routes in the tree diagrams of the questions. For example, considering the question in Figure 2, only 70% of the pupils were able to identify the possible routes to tree B (i.e. the routes C → U, C → Z and C → E). On the other hand, only 55% of the pupils managed to identify the route to tree A. In general, the percentages of the correct responses to the eight questions of the first scale 'ranged from 49% to 84%. Across the eight questions of the scale, the average score was only 5.2 with a standard deviation of 2.1 marks.

Questions of the second scale (What is the most likely outcome?):

The success rate of the pupils on this scale was comparable to their success rate on the first scale. For example, 68% of the pupils indicated correctly that reaching tree B (in the question of Figure 2) was more likely than reaching tree A. A percentage of 61% managed to indicate correctly that, in the question in Figure 1, the boy was more likely to enter room B rather than room A. On average, the pupils answered 2.6 of the 4 questions correctly.

Questions of the third scale (What is the probability of an event?):

A very small number of pupils managed to correctly identify the probability that events would happen. For example, on the question in Figure 1 only 2 pupils succeeded in computing the probability of the boy entering room A and only 5 pupils succeeded in computing the probability of the boy entering room B. However, 68% of the pupils answered correctly that the boy was more likely to enter room B rather than room A. Similarly surprising results were also found in other questions: the pupils could not

correctly compute the probability that events would happen but they gave correct responses to the 'What is the most likely outcome' questions.

A closer inspection of the responses of the pupils revealed that they used a simplistic approach to the questions of this scale, which coincidentally allowed them to give a correct response. For example, in the question of the Figure 1, four routes end in room B and three routes end in room A. This helped 314 pupils to answer that the probability of the boy entering room A was 3/7 and the probability entering room B was 4/7, therefore the boy was more likely to enter the room B rather than room A. Similar results were found for other questions.

The 'Explain Why' question:

Only one question of this type was included in the test. This was the last question of Figure 2. The pupils were allowed to express their responses in an open way. Their responses were aggregated into groups and are presented in Table 2.

Category of response	N	%
There are more routes leading to tree B	472	60.8
The route [a specific route] is straight/shorter	125	16.1
Tree A has more fruits than tree B	6	0.8
It is easier to follow a single route [to tree A]	6	0.8
Other answers	63	8.1
No answer given	104	13.4
Total	*776*	*100.0*

Table 2: A categorization of the responses to the 'Explain Why' question of Figure 2

According to the table, 61% of the pupils explained that they decided upon the most likely event using a probabilistic mode of reasoning ('There are more routes leading to tree B'). A small percentage of the pupils (13.4%) did not give any answer. A rather low 8% of the sample gave explanations that could not be categorized easily ('Other answers' category). The rest (17.7%) of the sample explained that they decided upon the most likely event based on subjective elements like the length of the routes, the number of fruit on the tree etc.

The role of grade, age and gender

Questions of the first scale (What are the possible routes?):

No statistically significant difference was found between the performance of boys and girls on this scale ($mean_{boys}$= 5.13; $mean_{girls}$= 5.30; t=1.129; p=0.259). However, a statistically significant difference was found between the pupils of different grades (mean of 4^{th} Grade=4.59; mean of 5^{th} Grade=5.72; mean of 6^{th} Grade= 5.39; F=20.103; p<0.001). More specifically, the 5^{th} and 6^{th} Grade has statistically significantly higher performance than the 4^{th} Grade. The 6^{th} Grade has lower performance than the 5^{th} Grade but the difference is not statistically significant.

The Pearson correlation between the age (measured in months) and the performance on the first scale was not found to be statistically significant (N=729, r=0.04, p=0.350). When, however, the correlation between the age and the performance was computed for each grade individually, small but statistically significant and negative correlations

appeared for the 4th and 6th Grade (4th Grade: N=238, r=-0.183, p=0.005; 5th Grade: N=225, r=-0.127, p=0.057; 6th Grade: N=266, r=-0.184, p=0.003). The correlation for the 5th Grade was small and negative but marginally non-significant.

Questions of the second scale (What is the most likely outcome?):

No statistically significant difference was found between the performance of boys and girls on this scale ($mean_{boys}$= 2.56; $mean_{girls}$= 2.55; t=0.075; p=0.940). However, a statistically significant difference was found between the pupils of different grades (mean 4th Grade=2.04; mean 5th Grade=2.89; mean 6th Grade=2.77; F=30.258; p<0.001). More specifically, the 5th and 6th Grade had, statistically, significantly higher performance than the 4th Grade. The 6th Grade had lower performance than the 5th Grade but the difference was not statistically significant.

The Pearson correlation between the age and the performance on the second scale was statistically significant and positive (N=729, r=0.14, p<0.001). When, however, the correlation between the age and the performance was computed for each individual grade, small, statistically insignificant and negative correlations appeared for all Grades (4th Grade: N=238, r=-0.112, p=0.086; 5th Grade: N=225, r=-0.107, p=0.109; 6th Grade: N=266, r=-0.107, p=0.083).

Questions of the third scale (What is the probability of an event?):

The performance of the pupils was so poor (fewer than 5 pupils answered correctly in each case) that no further investigation by gender or age was meaningful. The percentage of correct responses was usually between 0% and 1%.

Alternatively, it was attempted to identify the pupils who employed the more simplistic approach described in the section 'Overview of the results'. One mark was awarded to the pupils whenever they employed the simplistic approach to answer a question. The result was a score which measured the extent to which the pupils used the simplistic approach to respond to the questions of this scale. A larger score meant that the pupils used this technique more. The average score was 1.44 indicating that the 'average' pupil responded to one and half questions (out of four) using the simplistic approach.

No statistically significant difference was found between the performance of boys and girls on this scale ($mean_{boys}$=1.51; $mean_{girls}$=1.37; t=1.243; p=0.214). However, a statistically significant difference was found between the pupils of different grades (mean 4th Grade=0.66; mean 5th Grade=1.92; mean 6th Grade=1.76; F=54.53; p<0.001). More specifically, the 5th and 6th Grade had statistically significantly higher performance than the 4th Grade. The 6th Grade had lower performance than the 5th Grade but the difference was not statistically significant.

The Pearson correlation between the age and the performance on the third scale (using the simplistic approach) was statistically significant and positive (N=729, r=0.242, p<0.001). When, however, the correlation between the age and the performance was computed for each grade individually no statistically significant results were found.

The 'Explain Why' question:

Table 3 indicates that there was a significant tendency of the 4th Grade pupils to give more subjective and fewer probabilistic responses. On the contrary, the 5th and 6th Grade pupils gave significantly fewer subjective but significantly more probabilistic responses.

Grade	Various responses	There are more routes leading to tree B	Subjective responses (e.g. the number of fruit)	No response given
4th Grade	11%	44.3%	30.7%	14%
5th Grade	5.8%	71.1%	9.1%	14%
6th Grade	7.4%	67.8%	12.6%	12.2%

Table 3: A classification of the 'Explain Why' responses by Grade

Within the Grades, however, no difference to the average age of the pupils who gave each of the responses was recorded.

DISCUSSION AND CONCLUSION

It was found that even the pupils who had met tree diagrams and multiplication of probabilities in the past could not see their relevance to the questions of the test. This agrees with previous research (Green, 1987). In this study, many pupils employed a rather simplistic technique to answer the questions. They counted the number of all possible routes to any destination to formulate a sample space; they counted the number of favorable events (the possible routes to the target); they computed the probability to reach a destination by dividing the favorable events by the sample space size. Although this technique was, in principal, incorrect, for the pupils it was a very reasonable solution.

Another one of the significant findings of the research was that one in six pupils' probabilistic thinking was governed by subjective elements. Those pupils decided upon the most likely event by taking into consideration factors like the length of the route or even the number of fruit on the trees in the figure! Approximately 30% of the 4th Grade pupils (who did not have much exposure to these kind of questions) explained their response in the question in Figure 2 using subjective elements. The percentage for the 5th Grade pupils was only 9%, but again, this group had these types of questions in their mathematics books. Finally, only 12% of the 6th Grade pupils responded in a subjective way although they had never encountered this type of question before. All in all, it seems that teaching and age can significantly improve pupils' probabilistic thinking.

Gender did not seem to have much effect on the responses of the pupils. Age correlated significantly with the score of the pupils on the scales. It was striking that age, within the grades, correlated significantly but negatively with the ability of the pupils to identify the possible routes of the tree diagrams. No obvious explanation for this seems to exist.

This study investigated the interaction of primary school pupils with questions that demanded interpretation of tree diagrams and multiplication of probabilities. Overall, the results agree with previous research (Green, 1987; Ojeda, 1999). Still much remains to be researched, especially in the context of primary education in Cyprus. It is important to further investigate the failure of the pupils to realize the relevance of the tree diagrams and of the multiplication of probabilities when answering the questions in the test. It is also important to investigate the conditions under which some pupils' thinking is affected by subjective elements. Finally, it is important to study whether the probabilistic thinking of the same pupils is influenced by subjective elements when confronted with probability questions of a different nature e.g. when picking coloured marbles randomly from a bag. Semi-structured interviews with pupils are planned as a next step.

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